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SELECTED ASPECTS OF STABILITY OF THIN-WALLED STEEL STRUCTURES WITH CLEARANCES AND INITIAL IMPERFECTIONS

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ABSTRACT: This paper addresses the issue of stability of steel structures with the emphasis on their sensitivity to initial imperfections and clearances. The main aim is to performed an indication as the state of stress, displacement and the critical load. The paper presents theoretical studies involving linear and nonlinear behaviour of thin-walled steel structures with a special reference to the interactive buckling, initial geometric imperfections and clearances which is in the scope of the modern approach to design. For this purpose author proposed different structural models composed of rigid bars where strains are concentrated in connecting elastic joints These models enable to derive nonlinear algebraic equilibrium equations which strictly describe pre- and post-buckling behaviour of structures with various combinations of imperfections and clearances. Numerous examples demonstrate variable types of structural response depending on the modes and amplitudes of imperfections in relation with clearances.

Keywords: thin-walled structures, stability, geometric imperfections, clearances.

1. INTRODUCTION

Aesthetic and economic reasons have caused the rapid development of light-weight, thin-walled steel structures. It was possible thanks to modern highly automated cutting, drilling welding and corrosion protection and optimal structural design of elements, which can be easily manufactured and assembled. It must be underlined that two special aspects in design of light-weight steel structures must be seriously taken into account, namely fire protection and protection against local and global instability. In fact this steel structures are apt to fail by loss of global stability, which is the consequence of their high slenderness. Moreover they demonstrate tendency to local instability phenomena which often appeared at a similar load level as global one. The case when two or more different modes in stability and dynamic analyses are associated with the same or similar eigenvalues is termed a bimodal or multimodal solution. Designers' concern is that these solutions are very sensitive to imperfections. Moreover design engineers often face problems of slackened structures e.g. with slotted connections. This type of connection can appear at supports or in joints between substructures and can take a form of deliberately introduced expansion joints or can appear as clearances resulting from cyclic loading or manufacturing tolerances. Theory of slackened structures has been elaborated by Gawęcki (Ref. 1). He also demonstrated that clearances can drastically influence the structural response. This influence can be strongly disadvantageous, however, when the clearances are properly designed, then they can improve the structural response. Various problems of static and dynamic response of slackened structures were next studied in a number of papers. Influence of clearances on stability response of structures was discussed by Rzeszut (Ref. 2). The non-linear dynamic response of two-DOF systems with clearances was investigated by Kranjcevic et all (Ref. 3). The authors analysed the stability of steady state forced vibrations using the finite element in time method. The influence of slotted bolted connection on dynamic response of steel frame structures has been studied by Law (Ref. 4). The authors proposed the analytical model of slip with friction appearing in commonly used slotted bolted connections in steel frame structures. Behaviour of bolted connections with slotted holes was also investigated by Wald (Ref. 5), who analysed the influence of slotted holes in bolted connections on the stiffness and deformation capacity of structures and took into consideration the erection tolerances.

In the present paper, the stability of structures with clearances is studied in geometrically linear and non-linear ranges. The phenomena are illustrated using simple analytical models, where the clearances are consider as a translation or rotation gaps. Next, using FEM models, the analysis is carried out on real engineering structures with clearances, which take a form of slotted connections. Particular attention is focused on local and global buckling modes, which can appear in interactive buckling. It can result in excessive sensitivity to imperfections and often leads to drastic decrease of the limit load and unstable post-buckling behaviour discussed by Szymczak (Ref. 6). Therefore, the post-buckling analysis is carried out employing shell elements and introducing initial geometric imperfections. The imperfections are described in the form of the series of eigenmodes, where a limited number of most critical eigenmodes is used and the error minimization in this approximation is performed using the method proposed by Rzeszut (Ref. 7). Economic and safe design must be based on a reliable structural analysis where all essential imperfections and clearances are taken into consideration. On the other hand a numerical model of the actual structure cannot be too developed and complicated. Otherwise computer time would be too great. Of course, in order to demonstrate these phenomena nonlinear stability problems must be formulated and solved.

2. RANGE OF APPLICATION

Thin-walled steel structures can be used in the form of shells and bars such as facades, purlins, wall rails, scaffolds and main bearing capacity members of structural systems in form of frames or trusses. In the group of shell elements, can be distinguish trapezoidal sheets (Fig. 1a), sandwich panels (Fig. 1b), panel systems and façade cassettes (Fig. 2a and b). They provide high flexibility of solutions and ease and speed assembly, and in combination with the appropriate finishing elements complete façade solution. Trapezoidal sheets with a low profile (14 mm - 60 mm) are used to cover façade and roof surfaces, while sheets with a higher profile (135 mm - 160 mm), due to their strength, are used as supporting structures with considerable spans, on the roofs of large industrial, commercial and service buildings as well as for the production of composite floors. Widespread use on the walls and roofs is made of sandwich panels consisting of two steel facings with a thickness of 0.5 mm and a structural-insulating core made of polyurethane, polystyrene or mineral wool. The high quality of the façade's performance distinguished by an interesting visual effect and a variety of colours can be obtained by using panel and cassette systems. The panels are made of sheets with a thickness of 0.5 mm to 0.7 mm. They can reach a maximum span of up to 8 meters, while the minimum span may not be less than 1 meter. The cassette systems use galvanized sheets with a thickness of 1 mm to 1.5 mm or aluminium sheet with a thickness of 1.2 mm.

a) b)

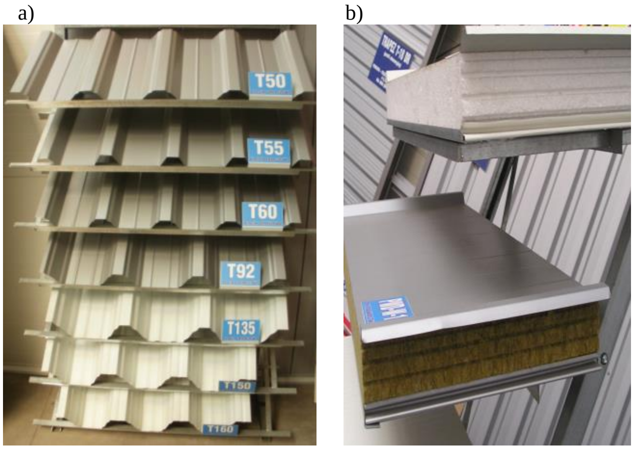

Fig. 1 Covering walls and roofs: a) trapezoidal sheets, b) sandwich panels (Blachy Pruszyński)

a) b)

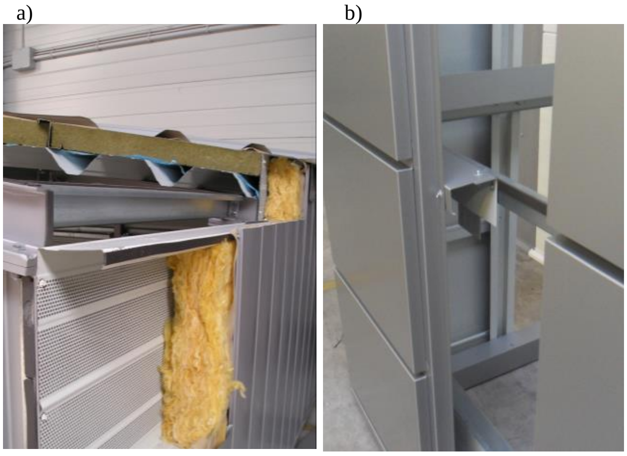

Fig. 2 Construction details of the façade made of steel thin-walled cross-sections: a) facade panels, b) cassettes (Blachy Pruszyński)

In the range of bar elements, cold-formed cross-sections of Z, C and Σ type are use. They are manufactured from galvanized steel strip with a thickness from 1.50 mm to 3.00 mm. The typical use of this sections usually includes the construction of wall cladding in the form of wall rails and roof covering in form of purlin or high storage racks. On the purlins, sections with Z and C cross-sections are usually used in a single-span or multi-span layout. They constitute also structure of floor and frame systems. Usually they are used in form of double cross-sections because of the weak geometric characteristics with respect to torsional resistance of a single cold-formed cross-sections (Fig. 3). Recommended spacing of the main load-bearing frames should be a multiple of the module 3.00 m or 1.50 m.

a) b)

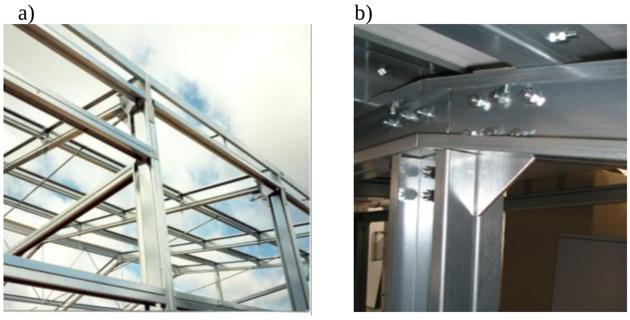

Fig. 3 Frame systems: a) Practa - -double Σ profiles, b) Blachy Pruszyński - double C profiles

3. THEORETICAL BACKGROUND

3.1 Initial geometric imperfections

Geometric inadequacies of metal structures called imperfections are inescapable features of a real construction and describe deviation from the ideal design characteristic. Geometric imperfections refer to deviation of the shape of the structure regarded as a whole or separately in the form of structural elements (bars, shells, connections). Technological imperfections result from manufacturing and assembly processes and generate a clearances in the construction mainly in the form of eccentricities and shifts. The standards for the design of metal structures EN 1993-1-1 (Ref. 8) describe design geometrical imperfections applied to the ultimate load analysis. They are introduced to the design model in the form of equivalent deformation or load (Fig. 4). According to the standard it is recommended to take into account the global imperfections of framework and bracings and local at the levels of individual elements. This division is not entirely accurate, since both single component and the whole framework system can have global and local geometric imperfections as well. In the global analysis the geometric imperfections are included in the static calculation model in the form of equivalent geometric configuration or by introducing a modified loads in static scheme. In the case of multi-storey framework the concept of global geometric imperfections is implemented by means of an equivalent geometric imperfection in the form of an initial sway imperfection, which includes the impact of the real imperfection of the initial column sway, eccentricity occurring in the joints during assembly process.

a) b)

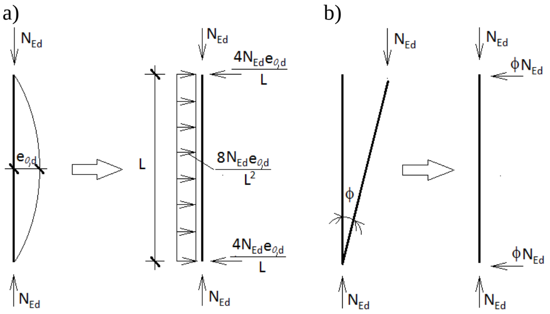

Fig. 4 Equivalent imperfections load system: a) bow, b) sway

On the other hand geometric imperfections are introduced into the numerical model by perturbation of the initial geometry. In classical approach it is assumed that imperfections have the form of an eigenmode associated with the lowest eigenvalue or in the form of linear combination of a few eigenmodes. This approach is in agreement with observations that eigenmodes represent the most dangerous shapes of imperfections. However, it is still an open question the choice of eigenmodes and scale parameters in the linear combination.

3.2 Clearances in thin-walled structures

In real engineering structures clearances can be deliberately introduced to compensate for unavoidable dimensional tolerances and thus to avoid random pre-stress during assembling. The clearances can also be introduced with the purpose to improve the structural response, e.g. in elasto-plastic regimes. In thin-walled structures the clearances can be formed by cyclic loads. Clearances of this type of are often observed in bolted connections of thin walled elements and usually they deteriorate the structural response. All types of clearances influence the structural response and therefore they should be accounted for in the structural analysis and design. Slotted connections are usually modelled by introduction bolts in slotted oval long or short holes. Nominal clearance of holes for screws and bolts is defined as the difference between the nominal size of the bolt and hole diameter. Static and dynamic analyses of slackened structures were discussed in the literature. Theory of slackened structures has been elaborated by Gawęcki. He also demonstrated that clearances can drastically influence the structural response. This influence can be strongly disadvantageous, however, when the clearances are properly designed, then they can improve the structural response, in particular in the elasto-plastic regime, where the shake-down plays essential role. In (Ref. 9) Gawęcki and Kuczma proposed an incremental formulation of a linear complementary problem (LCP) for solving elastic-plastic unilateral contact problems in slackened systems. In (Ref. 10) they analysed slackened skeletal structures by mathematical programming.

Influence of clearances on structural stability is still an open question.

As mentioned before, the slack in real engineering constructions is an unavoidable consequence of the occurrence of geometrical tolerances and most often occur in the nodes and supports of the structure. Therefore, to understand the nature of the clearance in a node or support, the relationship between the reaction at the support R_n and the displacement u_n should be analysed. In the case of a construction without clearance, the behaviour of the support is described by a model of a spring support with an unlimited elastic range (Fig. 5a). The value of the reaction R_n depends on the stiffness $k_{n\Delta}$ of the support and can be calculated using the following relationship:

$$R_n = k_{n\Delta} u_n , \tag{1}$$

where: $k_{n\Delta}$ means stiffness of the support.

a) b)

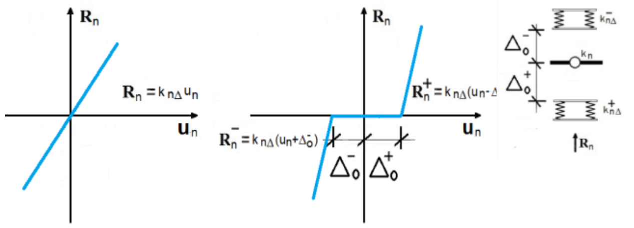

Fig. 5 Graphical representation of reaction forces in support with clearances (Ref. 11)

In the case of structures with clearances Δ_o, a support model shown in Fig. 5b should be used. In order to describe the relationship between displacement u_n and reaction R_n, two stages are considered: the first, when there is clearance between the structure and the support, and the second, when as a result of the increase of displacements the right (Δ_o^+) or left side (Δ_o^-) clearance is removed. Then the reaction at the support can be described by the following equations:

$$\begin{aligned} R_n &= k_{n\Delta}^- \cdot \left(u_n + \Delta_0^-\right) && \text{dla} \quad u_n \le \Delta_0^- \\ R_n &= 0 && \text{dla} \quad \Delta_0^- < u_n < \Delta_0^+ \\ R_n &= k_{n\Delta}^+ \cdot \left(u_n - \Delta_0^+\right) && \text{dla} \quad u_n \ge \Delta_0^+ \end{aligned} \tag{2}$$

3.3 Stability analysis

In the classic stability theory, the basic task is to determine bifurcation points or limit points. The bifurcation point can be determined on the basis of the solving of the eigenvalue problem, which results in the information on the critical load value and the normalized eigenvector. On the basis of the critical load value, only the possibility of a multi-path equilibrium is determined, but no information about the post-critical behaviour of the structure is obtained. Therefore, it is extremely important to determine the type of the bifurcation point. For this purpose, an analysis of post-critical states should be carried out by determining higher order variations of potential energy. Hence, a stable and symmetrical bifurcation point can be distinguished when the following conditions are met:

$$\delta^2\Pi = 0,\ \delta^3\Pi = 0 \text{ oraz } \delta^4\Pi > 0 \tag{3}$$

and an unstable bifurcation point when:

$$\delta^2\Pi = 0,\ \delta^3\Pi = 0 \text{ oraz } \delta^4\Pi < 0 , \tag{4}$$

in addition, when:

$$\delta^2\Pi = 0,\ \delta^3\Pi > 0 \text{ oraz } \delta^4\Pi < 0 , \tag{5}$$

the bifurcation point is asymmetrical. It is worth to note that the achievement of the bifurcation point should not always be associated with the mechanism of collapse of the structure. Stability equilibrium can take a more complex form when there is more than one minimum of potential energy. Stability of thin-walled members with imperfection and clearances should be performed based on non-linear studies. The non-linear equilibrium equation can be written in the incremental form:

$$(\mathbf{K}^{O} + \mathbf{K}^{G} + \mathbf{K}^{U})\Delta\mathbf{U} = \Delta\mathbf{P} \tag{6}$$

where $\mathbf{K}^{O}$ is the small-displacement stiffness matrix, $\mathbf{K}^{G}$ is the initial stress matrix, and $\mathbf{K}^{U}$ is the displacement stiffness matrix, $\Delta\mathbf{U}$ is the vector of displacement increments, $\Delta\mathbf{P}$ is the increment of the external load vector. The Riks method is used to solve Eq. (6), for both stable and unstable post-buckling responses.

The initial geometric imperfections are introduced by perturbations in the "perfect" geometry in the form of series of most critical eigenmodes. The vector of imperfections **u** is introduced in the form:

$$\mathbf{u} \cong \tilde{\mathbf{u}} = \left[\tilde{u}_r\right] = \left[\sum_{i=1}^{n} \alpha_i U_{ir}\right]^T = \mathbf{U}^T \boldsymbol{\alpha} \tag{7}$$

Where: "r" is a consecutive number of displacement in FEM model, "n" denotes total number of eigenmodes used to represent the imperfection in the approximation, "i" is a consecutive number of eigenmode, $\mathbf{u} \in \mathrm{R}^{\mathrm{N}}$ represents node coordinate perturbation vector of imperfect geometry, α_i is the scale factor associated with the i^{th} buckling mode. The eigenvectors **U** have been computed by solving the linear eigenvalue problem:

$$(\mathbf{K}^{O} + \lambda\mathbf{K}^{G})\mathbf{U} = \mathbf{0} \tag{8}$$

where λ is the load multiplier and eigenvector **U** represents the buckling mode shapes. In Eq. (8) the proportional loading and linearization of the pre-buckling state was assumed. The critical buckling loads are $\lambda_i^{cr}P$, where P is the reference load (the base state).

4. EXAMPLES OF STRUCTURAL ANALYSIS

4.1 Stability analysis on model structures

To illustrate the most typical stability response of structures with clearances and make possible the comparison with the response of the respective structures without clearances, two 1-DOF model structures will be analysed (Ref. 11). First model structure is composed of a vertical, perfectly rigid column, supported by a skew bar, which is linear elastic with the total longitudinal stiffness k_1 [N/m]. Both, the column and the skew bar have perfect hinges at the bottom supports and they are interconnected also with a perfect hinge. At the upper side of the column there is an additional elastic support with the horizontal clearance Δ_o. It is assume that in the state of large deformation the reaction force at the top of column remains horizontal. The initial geometric parameters are: lengths of column L and skew bar L_1, a is the distance between supports and hence the angle $\alpha = arctg(a/L)$.

a)	b)

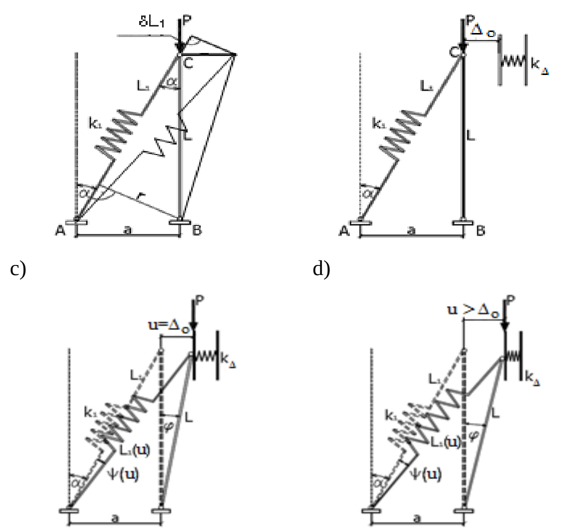

Fig. 6 Model structure: a) primary structure, b) clearance is open, c), d) clearance is closed (Ref. 11)

During the deformation the displacement parameters appear: horizontal displacement of the upper hinge Δ, rotation of the column φ and skew bar rotation Ψ. These displacement parameters can be computed from one control parameter Δ using the relations:

$$\sin\varphi = \Delta/L \quad \text{and} \quad tg(\alpha + \Psi) = (a+\Delta)/(L\cos\varphi). \tag{9}$$

The actual length $L_1(\Delta)$ of the skew bar and the distance $r(\Delta)$ of the point B are:

$$L_1(\Delta) = \frac{a+\Delta}{\sin[\alpha+\psi(\Delta)]} \qquad r(\Delta) = a\cdot\cos[\alpha+\psi(\Delta)] \tag{10}$$

Again, two stages in structural response are observed . The first one, when the clearance is open $|\Delta|<\Delta_o$, and the critical load equilibrium eq. of moment w.r.t. support B leads:

$$P\cdot\Delta = S\cdot r(\Delta) \text{ where } S = k_1[L_1(\Delta) - L_1] \tag{11}$$

For small Δ (see Fig. 6a):

$$\delta L_1 = \Delta\cdot\sin\alpha \text{ and } r = a\cdot\cos\alpha = L\cdot\sin\alpha \tag{12}$$

Introducing Eq. (12) into Eq. (11) one obtains a linear stability problem with:

$$P^{cr} = k_1\cdot a\cdot\sin\alpha\cdot\cos\alpha = k_1\cdot L\cdot\sin^2\alpha \tag{13}$$

The post-critical path P-ϕ can be calculated as follows:

$$P = \frac{1}{\Delta}\left(L_1(\Delta) - L_1\right)\cdot k_1\cdot a\cdot\cos[\alpha+\psi(\Delta)] \tag{14}$$

The second stage appears when the clearance is closed $|\Delta|\geq\Delta_o$ (Fig 5b, c) and the structural stiffness k_2 contributes to the bearing capacity. Then the load-displacement relation can be obtained from the balance of momentum of the column with respect to the bottom hinge B:

$$P = \frac{1}{\Delta}\left(L_1(\Delta) - L_1\right)\cdot k_1\cdot a\cdot\cos[\alpha+\psi(\Delta)] + \frac{\Delta-\Delta_o}{\Delta}k_2 L\cos\varphi \tag{15}$$

The equilibrium paths P- $\bar{\Delta}$ are presented in Fig. 7. Non-dimensional axes are used: displacement $\bar{\Delta} = \Delta/L$ and critical force $\bar{P}^{cr} = P/P^{cr}$, where P^{cr} follows from linear solution (13). support ($\Delta_o \to\infty$) in linear and nonlinear stability analysis, respectively. Plot 2 illustrates the response of the same structure, but with the elastic support without the clearances (Δ_o=*0*). It means that the additional support with the structural stiffness k_2 is active from the beginning. Plots 2 a, b, c, illustrate the influence of increasing values of clearances on stability response of the structure. Elastic support is switching on with delay, which is the function of the value of the clearance. It is well seen that all plots converge asymptotically to the equilibrium path of structure with the additional support without a clearance. The model under consideration demonstrates unstable post-buckling behaviour, therefore the analysis for negative Δ can provide next information on the stability of the structure. In Fig. 8 and 9 the influence of the stiffness of the horizontal support on the stability response is presented.

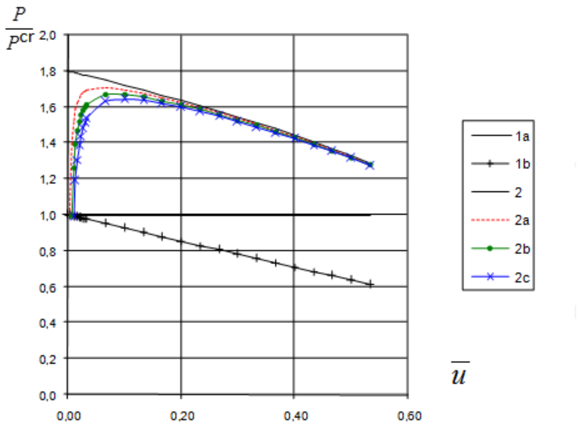

Fig. 7 Stability response without and with clearances (η=0.4) (Ref. 11)

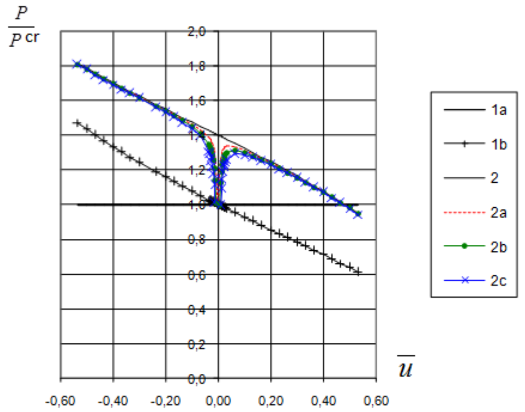

Fig. 8 Asymmetrical unstable bifurcation point η= 0.2, b) η= 1 (Ref. 11)

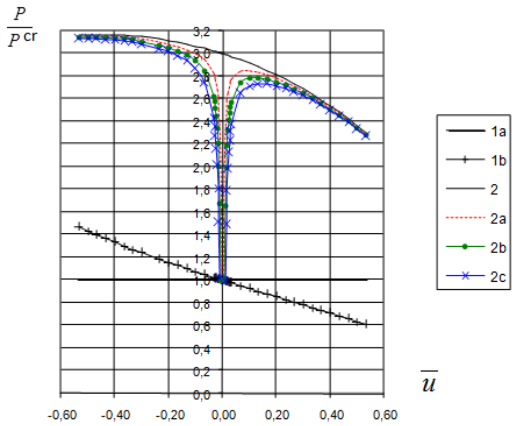

Fig. 9 Symmetrical and unstable bifurcation point, η=10 (Ref. 11)

The plots 1a and 1b refer to the primary structure without the additional support. We the non-dimensional stiffness coefficient $\eta=k_1/k_2$ is introduced. One can notice, that for a reasonable small value of η, the structure demonstrates the asymmetrical and unstable bifurcation point (Fig. 8). For a high value of η (Fig. 9), the bifurcation point changes form approaching to a symmetric and unstable point. It is well seen that, not only the amplitude of the initial clearances but also the flexibility of the support strongly influenced the structural response.

Second model consists of rigid bar with total length $2L$ an elastic hinge in the middle point, with the rotational stiffness k_1 [Nm/rad]. There is also a transverse elastic support in the middle point. This support has the stiffness k_2 [N/m] and the clearance Δ_o, whereas Δ denotes total transverse displacement of the middle point of the bar. Reaction forces at the end supports and at the middle support remain vertical also in the states of large deformation. Two stages of structural behaviour are observed. The first one for $|\Delta|<\Delta_o$, when the clearance is open (Fig. 10a) and the structure responses as a typical Euler column with the stable symmetric bifurcation point. The second stage appears when the clearance is closed $|\Delta|\geq\Delta_o$ and the structural stiffness k_2 contributes to the bearing capacity.

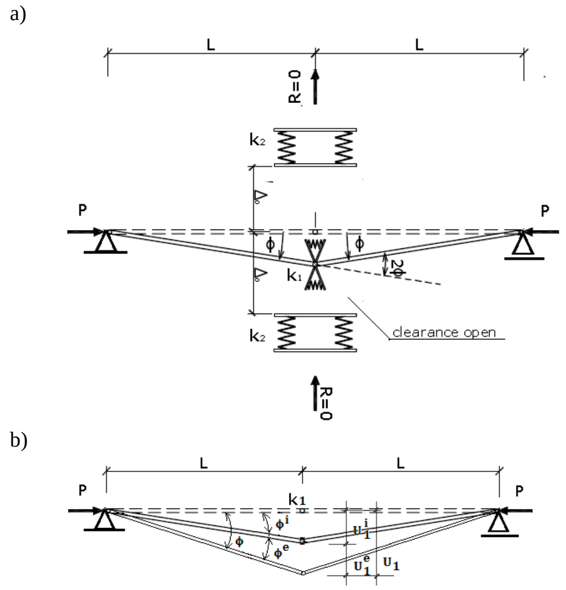

Fig. 10 Model structure: a) with internal supports with clearances at top and bottom side, b) initial geometrical imperfections (Ref. 11)

Initial imperfection in the form of initial displacement illustrated in Fig. 10b induces initial rotations of bars and respective hinge rotations ϕ. These deformations are kinematic admissible therefore stress free. Then elastic deformations appear and induce moments in elastic hinge.

In initial state when P = 0, the geometric relations take the form:

$$\sin\phi^i=\frac{u_1^i}{L} \qquad hence \qquad \phi^i=asin\frac{u_1^i}{L} \tag{16}$$

In current state, when $P > 0$ we have:

$$u_1=u_1^i+u_1^e \quad \text{and} \quad \phi=\phi^i+\phi^e \tag{17}$$

Now, the non-linear equilibrium equation takes the form:

$$k_1\phi^e=P\left(u_1^i+u_1^e\right) \quad \text{or} \quad 2k_1\left(asin\frac{u_1}{L}-asin\frac{u_1^i}{L}\right)=P\left(u_1^i+u_1^e\right) \tag{18}$$

with unknown u_1^e.

Next, the structural response for various magnitude of imperfections is analysed . In Fig. 11 the influence of initial geometrical imperfection on model structure is presented. One can notice that structure responses as a typical Euler column with the stable symmetric bifurcation point. Set of plots from red to green, illustrate the influence of increasing values of initial geometrical imperfection on stability response of the structure. It is observed very interesting phenomenon that for the imperfect structure, the stable equilibrium path is accompanied by second unstable solution. The influence of imperfections combined with clearances is presented in Fig 12. The equilibrium paths are calculated for model structures with initial geometrical imperfections u_2^i=0.01L and 0.07L and clearance Δ_o=0.2L. The dash line represents the reference structure without internal supports, the solid lines correspond to the structure with various support stiffness $k_2^1<k_2^2<k_2^3$. When clearance is open all plots are identical then clearance is closed then diversified plots are observed. One can notice that increasing value of support stuffiness results in displacement reduction.

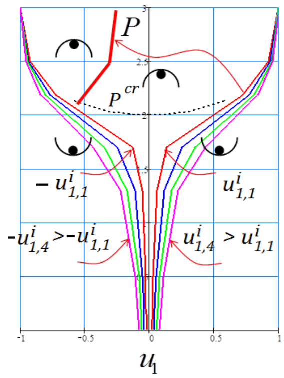

Fig. 11 Influence of initial geometrical imperfection on model structure (Ref. 11)

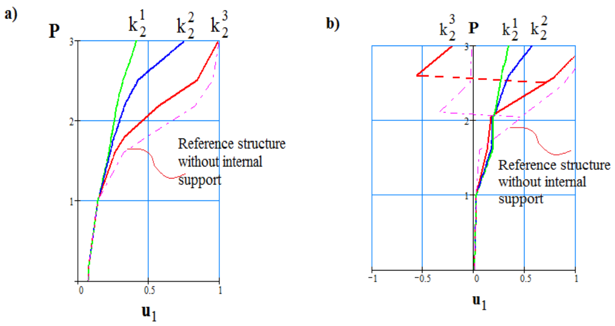

Fig. 12 The equilibrium path for model structures with initial clearance and various support stiffness for two geometrical imperfections a) u_2^i=0.07L , b) u_2^i=0.01L (Ref. 11)

More interesting is that there is an interaction between imperfections, clearance and support stiffness. The example presented in Fig. 12a pertains the reference structure which responses in form of stable post-buckling path as well the structure with clearances also demonstrates stable equilibrium path for various support stiffness. Whereas, the example presented in Fig. 12b corresponds with case when reference structure beside stable equilibrium path demonstrates unstable post-buckling behaviour. Now, introducing the initial clearance results in different structural response. For the sufficient high support stiffness k_2^1 and k_2^2 the structure responses as typical Euler column, while for the relatively small k_2^3 bifurcation point appeared and stable or unstable post-buckling behaviour is observed. This proves that the structure under consideration is sensitive to both the imperfections and clearances.

4.1 Numerical analyses

Numerical analyses were conducted on the example of the bar which consist of 2Σ thin-walled cross- sections connected by means of bolts (Ref. 11). In numerical model bolts were modelled in various ways. In order to avoid very fine mesh in bolts and in the surrounding area, which is rather unacceptable in the nonlinear analyses of total members, the action of bolts will be modelled by introducing constraints on displacements of FEM nodes, that are located in points A and B, where the axis of a bolt crosses the middle planes of the web of Sigma sections (Fig. 13). The first model of connection is based on the assumption that the bolt is flexible and preserves only the constant distance between points A and B. Rotations are free. The name "*Link"* will be used. The second model will be called a "*Tie"*. In this model also the rotations are constrained. The last model referred to as a "*Slot"*, accounts for the clearances in the connection. It is assumed that the point A is fixed, whereas the point B may move along the slot with a predefined length. Two direction of the slot are analysed, namely horizontal and vertical. The "*Slot"* with and without friction is distinguished, too.

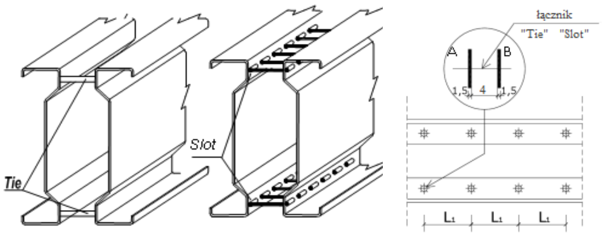

Fig. 13 Connectors of cold formed double SIGMA elements (Ref. 11)

In the study special attention is focused on interactive buckling. Therefore there were extracted a few first eigenvalues which correspond to global and local buckling, not only the lowest value of λ_1, which is usually of designers' interest. In Fig. 14 the non-dimensional critical stress against column slenderness ratio L/i_z is plotted, for double sections connected by connector type "Tie". For the slenderness ratio L/i_z<100, the values of critical stresses corresponding to local and global buckling mode are close to each other. In this case the designers should be rather conservative, because they can face the interactive buckling phenomena.

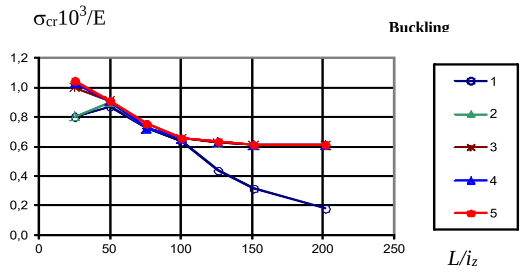

Fig. 14 Critical stress for column made of 2Σ profile connected by connector type "Tie" (Ref. 11)

The nonlinear study was carried out for a beam made of 2Σ profile of 4 m span (L/i=100) with a spacing of connectors equal to 1.0 m. This is the case, where critical stresses, associated with local and/or global buckling modes, have similar values for several eigenmodes.

In Fig.15 the plots of the load factor λ versus "arc length parameter" in Riks method are shown.

The plots refer to global (*g*) and local (*l*) shape of imperfections and different numerical models of connectors. The maximum amplitude of initial global imperfections was 4 mm and for local imperfections was 2.04 mm. Plots 1*l* and 1*g* illustrate the equilibrium paths for connectors' model "Tie" for local (*l*) and global (*g*) shape of imperfections, respectively. One can notice that the introduction of local initial imperfection results in stable equilibrium path. However, the same structure with global initial imperfection demonstrates unstable post-buckling behaviour. In this model the maximum load capacity in relation to the λ^{cr} of the structure with perfect geometry appeared to be reduced by 11%. Plots 2 and 3 describe the load capacity obtained for the structures with clearances (numerical models of connectors „Slot") without and with friction, respectively.

The analysis demonstrates that introduction of the clearances in the way of "Slot" connectors results in unstable post-buckling behaviour for local (*l*) and global (*g*) shape of imperfections. The reduction of the load capacity in the case without the friction in relation to the perfect structure is around 18%. One can also notice that the type of connectors strongly affects the post-buckling behaviour.

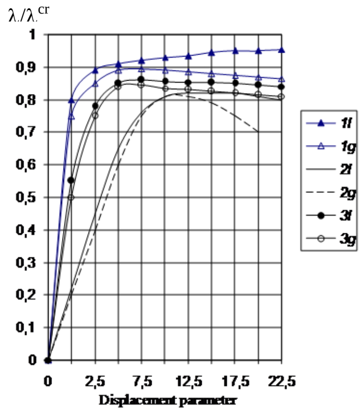

Fig. 15 Load proportionality factor of column made of double Σ cross section for global (*g*), local (*l*) shapes of imperfections (Ref. 11)

4. CONCLUDING REMARKS

In the paper the stability analysis of simplified model structures with clearances and initial geometrical imperfections is performed. Due to its simplicity the exact close form solutions is described for both linear and nonlinear stability analysis. To illustrate the most typical stability response of structures with clearances the comparison with the structure without clearances is carried out. It was shown, that interaction between initial clearances and initial geometric imperfections can strongly affect the structural stability response. When the reference structure is characterised by stable post-buckling behaviour, the structure with clearances also demonstrates stable equilibrium path, which converges asymptotically to the former one. And conversely, when the reference structure responses in form of unstable post-buckling path, then it demonstrates sensitivity to geometric imperfections and small initial clearances, moreover unstable post-buckling behaviour can be reached. This observation can be interesting and useful for practical engineers, because the experience and intuition with respect to initial imperfections can be used when treating the clearances. Numerical examples illustrate the importance of proper numerical modelling of connection. It is demonstrated that the type of connector, its flexibility and the way of connection strongly influences the critical load and post-buckling behaviour.

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